MATHEMATICS (US)

Paper 0444/13
Paper 13 (Core)

Key messages

Ensure answers are given to the required accuracy and avoid premature rounding in working.
Care needs to be taken with writing, particularly figures which are often not clear.

General comments

The standard of candidates' responses was generally very good. The majority appeared to have had sufficient time to complete the paper.

Revision should include basic skills as well as the advanced topics; many candidates could not handle basic place value in **Question 13(a)**.

The topic of functions did not appear to be well understood.

Comments on specific questions

Question 1

This measuring question was generally answered correctly. The most common error was 7.9.

Answer: 7.4

Question 2

The majority of candidates gave the correct answer.

Answer: 15

Question 3

This question was generally well answered.

Answer: 5

Question 4

The most common error was to work out the numerator but then not to divide by 7.

Answer: 0.7

Question 5

Many candidates knew they had to put a bracket into their answer and several were able to gain the mark. Others were unsure of how to deal with the first y and took it as the factor but then just wrote $2y$ or $-2y$ inside the bracket.

Answer: $y(1 - 2y)$

Question 6

This question was generally correct.

Answer: Sphere

Question 7

Many candidates knew to draw a vertical and horizontal line and scored both marks. A significant number also added diagonals.

Question 8

Many candidates showed a good understanding of dividing in a given ratio. The most common error was to divide 72 by 5 and then by 4, giving answers of 14.4 and 18. Some gave amounts not adding to 72.

Answer: 40, 32

Question 9

(a) Most candidates gave the correct answer. There were a small number who gave the answer 13.

(b) This part was also very well answered. The main error was 12 from $15 - 3$.

Answers: **(a)** –3 **(b)** 18

Question 10

While many candidates were able to give the correct simplification, there were a significant number who did not understand which sign applies to which term. A small number tried to factorise and gave their answer as $p(2-5)-q(1-3)$ or similar.

Answer: $-3p-4q$

Question 11

(a) Some candidates gave the correct answer. There appeared to be a lack of understanding of significant figures and many appeared to confuse the term significant figures and decimal places. 0.08 was a common answer.

(b) This part was less well answered with many not giving place holders, leading to 10 as a common incorrect answer; the other common error was 10 100.

Answers: **(a)** 0.076 **(b)** 10 000

Question 12

Many candidates had followed the wording of the question and scored both marks.

Answer: $\frac{3}{8}$

Question 13

(a) This part was generally well answered. The most common errors were as a result of not understanding place value, as shown by answers of 5 200 007, 500 200 007, 5 207 000.

(b) This part was less well answered. Many made an attempt at using a power of 10 but it wasn't written in scientific notation, e.g. 81.3×10^{-4}.

Answers: **(a)** 5 000 207 **(b)** 8.13×10^{-3}

Question 14

Almost all candidates understood the term factor. Very few listed any numbers which were not factors of 30. The most common reason for not scoring both marks was due to not listing all the factors; usually 1 and 30 were omitted.

Answer: 1, 2, 3, 5, 6, 10, 15, 30

Question 15

The majority of candidates scored both marks.

Answer: 0.27

Question 16

(a) Many candidates clearly understood the term slope but gave the answer as 4*x* rather than 4.

(b) This part was less well answered. A common error was to give the answer 6 rather than –6. Several candidates did not attempt to answer this part.

Answers: **(a)** 4 **(b)** –6

Question 17

Candidates generally understood what was required to answer this question and both marks were often scored.

Answer: 12

Question 18

Candidates generally understood what was required to answer this question and both marks were often scored.

Answer: $2\frac{1}{3}$

Question 19

(a) Many candidates were able to give the correct answer. Some clearly knew the median as the middle value but had not listed the values in order first, and gave the answer as 46.5 from 73 and 20, the middle values in the original list. Some correctly ordered the list but were unable to work out the mean of 20 and 23, giving the answer 22. A small number calculated the mean rather than the median.

(b) This part was less well answered. A common error again was not thinking about the order of the numbers and just calculating 54 – 16. Some who had correctly used the highest and lowest values wrote 95 – 7 on the answer line without resolving it.

Answers: **(a)** 21.5 **(b)** 88

Question 20

(a) Most candidates gave the correct answer.

(b) Many were able to give the correct answer and showed method. The most common error was to use 48 as one of the base angles and give the answer as 84.

Answers: **(a)** 100 **(b)** 114

Question 21

Many candidates were unable to give the correct answer. Several scored one mark for 360 ÷ 20. Many did not show any method.

Answer: 162

Question 22

(a) Many candidates were able to give the correct answer. A small number subtracted 7 rather than adding. A small number 'simplified' $3w - 7$ to –4w.

(b) Again many correct answers were seen in this part. Some made numerical slips when multiplying out the bracket. Some who gave the answer as a fraction had not cancelled to its simplest form.

Answers: **(a)** 13 **(b)** 0.7

Question 23

The majority of candidates didn't score full marks as they had not realised the units were different. The common answer was 60 from dividing 120 by 2.

Answer: 4000

Question 24

Compound interest did not appear to be well understood. Just attempting to find 2% of 5000 was seen.

Answer: 5100.5

Question 25

(a) Many candidates gave the correct answer. A small number reversed the co-ordinates.

(b) This part was also well answered. Those who had reversed the co-ordinates in part **(a)** generally did so in this part also.

(c) This part was less well answered than the previous parts. There were various combinations of ± 2 and ± 8.

Answers: **(a)** (5, 3) **(c)** $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$

Question 26

(a) (i) This topic was generally not understood.

(ii) The common error in this part was to calculate the slope.

(b) This part was not generally understood and a common answer was >5.

Answers: **(a)** No, with valid explanation **(b)** $0 \leqslant g(x) \leqslant 15$ **(c)** 1, 2, 3, 4, 5

MATHEMATICS (US)

Paper 0444/23
Paper 23 (Extended)

Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and show all necessary working clearly.
There are many ways to simplify the operations when a calculator is not available and candidates need to learn these and not use long multiplication and division when there are some simpler methods available.

General comments

The working in many cases was difficult to follow; candidates need to set it out logically. There is a need for some candidates to improve their arithmetical dexterity.

Comments on specific questions

Question 1

Almost all candidates answered this question correctly. However some attempted to multiply first rather than divide and were then more likely to make arithmetic errors.

Answer: 63

Question 2

Most candidates answered this correctly; the most common incorrect answer was $-y(2y)$.

Answer: $y(1-2y)$

Question 3

Most candidates answered this correctly, the most common error was in the division of 7 by 140.

Answer: 5

Question 4

Most candidates answered this correctly. Those who did not follow the correct order of operations gave an answer of 17.

Answer: −4

Question 5

The common error in this question was attempting $125 \times \frac{2}{3}$.

Answer: 25

Question 6

In part **(a)** the answer was usually correct; the most common incorrect answer was 500207. In part **(b)** the most common incorrect answers included 8.13×10^3 and 813×10^{-3}.

Answers: **(a)** 5000207 **(b)** 8.13×10^{-3}

Question 7

Most responses were correct; the most common incorrect answers included the wrong addition of the p and the q terms giving an answer of $-7pq$ or not adding the two similar terms together and leaving the answer as $p(2 - 5) + q(-1 - 3)$.

Answer. $-3p - 4q$

Question 8

Most candidates gave the correct answers in both parts. The most common incorrect answers were in part **(a)** 0.1, 0.077, 0.07600 and 0.08, and in part **(b)** 10100 and 10.

Answers: **(a)** 0.076 **(b)** 10000

Question 9

Most candidates gave the correct answer.

Answer: $\frac{3}{8}$

Question 10

Many correct answers were seen; the only error seen came from an incorrect first step of $3w = 32 - 7$.

Answer: 13

Question 11

This question was answered well, but a common incorrect start was $\pi r^2 - A = \pi rL$ or an incorrect attempt to divide by πr, leading to $\frac{A}{\pi r} = \pi r^2 + L$. A small number of candidates incorrectly tried to simplify a correct answer such as $\frac{A - \pi r^2}{\pi r} = A - r$.

Answer: $\frac{A - \pi r^2}{\pi r}$

Question 12

Many candidates managed to write $\sqrt{27}$ as $3\sqrt{3}$ but they could not simplify $\sqrt{243}$.

Answer: $6\sqrt{3}$

Question 13

Most candidates were unable to do this question and answers were varied.

Answer: 60, 300

Question 14

Very few candidates gave the correct answer for the period. Many gave 5 as the amplitude but this was seen for the period also.

Answer: [Amplitude =] 5 [Period =] 90

Question 15

The working in this question was not always logically set out. It was common to see 108 divided by 20 rather than multiplied by it. Many used 360 rather than 3600 for the seconds in an hour and some used 100 metres in a kilometre. It was also common to see $20 \times 30 = 60$ clearly written down.

Answer: 600

Question 16

Part **(a)** was usually answered correctly with an occasional *w* as an incorrect answer. Part **(b)** was not answered as accurately as part **(a)** with the most common incorrect answers being $3w^9$ or $9w^9$. A few left their answer in brackets.

Answer: **(a)** $\frac{1}{w}$ **(b)** $27w^9$

Question 17

The most common errors were using inverse proportionality and writing x^2 instead of $\sqrt{x}$. Some forgot to square root 25 when calculating the required value.

Answer: 10

Question 18

Most candidates knew the correct method and the most common error was to write $x^2 + 1$ for the denominator after writing $x(x + 1)$.

Answer: $\frac{1}{x(x+1)}$

Question 19

Many candidates calculated the perimeter correctly and found the correct values for *p* and *q*.

Answer: [*p* =] 12 $[q =] \frac{12}{5}$

Question 20

Many candidates could use the quadratic formula competently but they were unable to simplify $\sqrt{20}$. Hence many obtained the answer to *a* but not the answer to *b*.

Answer: [*a* =] 1 [*b* =] 5

Question 21

The main error seen in this question was that some candidates thought that the values of x lie between 0 and 8 so that $0 \leqslant x \leqslant 8$ was given as one of the answers and similarly, the values of y lie between 4 and 8 so that $4 \leqslant y \leqslant 8$ was given. There was also some confusion about whether to include the values on the line so some wrote the equality sign and some did not. Few attempted to find the equations of both lines.

Answer: $x \geqslant 0 \quad x + y \leqslant 8 \quad x + 2y \geqslant 8$

Question 22

Many candidates were able to factorise either the numerator or the denominator correctly, but not always both. Some attempted to cancel expressions which were not common factors. The most common incorrect approach was to attempt to cancel from the original fraction, for example $\frac{2x^2 - x - 1}{2x^2 + x} = \frac{-x - 1}{x}$.

Answer: $\frac{x-1}{x}$

Question 23

The major challenge for many candidates was identifying the correct angle to be found. A good proportion of the candidates were awarded credit for producing $\sqrt{\left(10\sqrt{2}\right)^2 + (10)^2}$, but then went on to use these in attempts to calculate the wrong angle, often attempting to find angle *PAB* rather than the required angle *PAC*. The most common correct approaches were working with sine or tangent, depending on whether they had tried to find *AP*.

Answer: 30

Question 24

Most candidates answered parts **(a)** and **(b)** correctly. In part **(c)** most of the candidates subtracted 1 correctly but they could not deal with the root, some finding the square root whilst others tried to find the fourth root, even before dealing with the 2.

Answers: **(a)** 9 **(b)** $\frac{5}{3}$ **(c)** $\frac{1}{16}$

Question 25

In part **(a)** some candidates left the partial factorisation as the final answer such as $x(p-1) + y(p-1)$. In part **(b)** many achieved the first step to $2(t^2 - 49\,m^2)$. However they did not see the expression in the bracket as a difference of two squares and often left the answer as a partial factorisation.

Answers: **(a)** $(x+y)(p-1)$ **(b)** $2(t+7m)(t-7m)$

Question 26

Those who understood the topic of vectors usually answered all three parts correctly. The main incorrect answers in part **(a)** were $\mathbf{a} + \mathbf{c}$ or $\mathbf{c} - \frac{2}{3}\mathbf{a}$ .In part **(b)** some candidates used $\mathbf{c} - \mathbf{a}$ for vector *CA* instead of $\mathbf{a} - \mathbf{c}$. In part **(c)** some candidates found vector *XP* but then could not work out the ratio whilst others gave the correct answer without any working being shown.

Answers: **(a)** $\mathbf{c} + \frac{2}{3}\mathbf{a}$ **(b)(i)** $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{c}$ **(ii)** 3 : 2

MATHEMATICS (US)

Paper 0444/33
Paper 33 (Core)

Key Messages

This paper required candidates to have a good knowledge across all of the key areas, number, algebra, shape and space and probability and statistics.
Candidates should show working to support their solutions.

General Comments

This paper covered a wide range of topics and candidates were able to evidence clear understanding across the syllabus. Candidates were able to complete the question paper within the given time and even when there were questions they found more challenging, candidates moved on to other questions they were able to answer. The standard of presentation was generally very good, but some candidates need to be careful that they write digits clearly and there is no ambiguity as to what they intend.

Comments on Specific Questions

Question 1

(a) Candidates were required to write all parts of 602 047 in words with no numbers included. Whilst the majority of candidates were successful, a variety of errors were seen. Some candidates made errors with place value, usually starting with six million, others included the word thousand twice, for example 'six hundred thousand and two thousand and forty seven'.

(b) (i) The majority of candidates gave a correct multiple of 14 with 28 being the most common correct response. Other correct responses included 14, 42 and 140. The most common incorrect answer was to give a factor of 14, usually 7.

(ii) This was well answered with almost all candidates giving the correct answer.

(iii) The majority of candidates cube rooted the given number correctly. Most candidates who did not score had incorrectly square rooted the given number.

(iv) Again the majority of candidates knew that $12^0 = 1$. The most common incorrect answer was zero.

(c) Many candidates answered this correctly and had approached the question by using factor tables or factor trees. Even if the correct LCM was not found, or if the HCF was found in error, candidates were often awarded one method mark for their tables or trees. There were a number of arithmetic errors seen as well as tables/lists with either too few or too many factors. Candidates who gave a larger multiple of 156 such as 312 were awarded one mark.

(d) Again many candidates answered this correctly or were able to score one mark for giving 2 or 3 as their final answer. Candidates who found the HCF in the previous part usually went on to find the LCM in this part.

(e) Candidates who used a factor tree were most successful in reaching the correct answer and the best answers were those that were written in index form, even though $2\times2\times2\times2\times3\times3\times3$ was also awarded full marks. Some candidates incorrectly assumed this part related to previous parts and others did not know how to approach the question.

Answers: **(a)(i)** Six hundred and two thousand and forty-seven **(b)(i)** any multiple of 14 **(ii)** 3136 **(iii)** 47 **(iv)** 1 **(c)** 156 **(d)** 6 **(e)** $2^4\times3^3$

Question 2

(a) (i) Nearly all candidates had the first two answers correct. Many scored the remaining two marks but often the third mark was lost by those who chose Mexico or Brazil rather than Bali. The last mark was frequently lost by those listing Caribbean and Brazil in reverse order or choosing the wrong combination of countries.

(ii) Many candidates answered this correctly but others were not sure how to approach the question. Those who recognised that Bali was one fifth of the pie chart were more successful than those who chose to find the number of people in every sector as these candidates occasionally forgot to add the 180. Some candidates scored full marks with no working shown.

(b) (i) The majority of candidates scored full marks on this question. The most common error was to find 70% of 450 correctly but then subtract this from 450 to get 135 as the cost of a child's ticket. Most of the candidates who did this managed to score one mark for the cost of the two adults' tickets.

(ii) A good proportion of candidates scored full marks on this question. Most others scored 1 mark for adding 2 hours 11 mins to 0929 correctly but then added the 1 hour time difference instead of subtracting it, thereby losing the second mark. Some scored 0 marks for giving the answer 1240 with no working shown.

(iii) This part was answered correctly by a large majority of candidates. A few lost all the marks by not reading the question properly and adding \$3.50 to \$2.15 then multiplying by 38.

(iv) Nearly all candidates scored full marks. Only a few multiplied 1335 by 17.8 rather than dividing.

Answers: **(a)(i)** 26, USA, Bali, Caribbean/Brazil **(ii)** 900 **(b)(i)** 1845 **(ii)** 1040 **(iii)** 85.2[0] **(iv)** 75

Question 3

(a) Only a minority of candidates were able to give the correct mathematical name for the shape. Parallelogram, rhombus and quadrilateral were some of the incorrect answers given.

(b) (i) A large majority of candidates knew the transformation was an enlargement and many gave the correct scale factor. Finding the centre of enlargement proved more challenging with many omitting it altogether or giving a vector. It was common for candidates to imply more than one transformation and to say the shape was translated/moved and enlarged. Candidates often described the shape as 'made bigger' rather than state 'enlargement', which did not score.

(ii) This part was not answered very well with many giving the transformation as a reflection. A significant number gave at least two transformations, usually including reflection or translation along with rotation. Vectors were often included as part of the answer and non-mathematical descriptions such as 'turn' or 'move' were very common.

(c) (i) Many candidates scored full marks for drawing the correct translation. Some scored one mark, usually for translating the shape correctly by the horizontal distance. A common incorrect answer was to translate the shape by $\begin{pmatrix}-4\\8\end{pmatrix}$.

(ii) Many candidates scored full marks for drawing the correct reflection. Some scored one mark for a reflection in the wrong vertical line or in the line $y = 2$.

(d) A small majority of the candidates found the correct area of the shape either by counting squares or by splitting the shape into a triangle and square. The correct answer was often seen with no working shown.

Answers: **(a)** Trapezoid **(b)(i)** Enlargement, scale factor 3, centre (–5, 0) **(ii)** Rotation, 180°, centre (0, 0) **(d)** 13.5

Question 4

(a) Nearly all candidates scored full marks for the correct scaled distance. Most of the rest scored one mark for measuring 6 cm.

(b) (i) This part was not well answered. Many incorrect angles were given including 75, 105 and 285 (from 360 – 75).

(ii) Very few candidates could describe the correct method to find the bearing and gave statements about using a protractor and measuring it. A common error was to subtract their previous answer from 360 (angles around a point) and not carry on to subtract this from 180 using interior angles. Others added 180 or subtracted their previous answer from 180. Many candidates did not give any response.

(c) (i) Although many candidates scored full marks, this part of the question was not very well answered. Many were able to score one mark usually for plotting *D* at a correct distance but with an incorrect bearing. Some neat and accurate diagrams to show the bearing and distance of *D* were seen but others had placed point *D* very inaccurately, sometimes measuring 100° from the line *AC* or a horizontal line at *C*. Others had read the wrong scale on their protractor and had the bearing at 80° rather than 100°. Candidates should be encouraged to represent the required point by a single dot rather than a large blob or just the letter *D*.

(ii) This part was reasonably well answered with many scoring full marks. A common error was to change the units of time from 1½ hours to 90 mins for the division which was not necessary. Others divided by 1.3 instead of 1.5.

(iii) The majority of candidates scored full marks in this part. Some made an error in the method with the division by 8 and multiplication by 5 the wrong way around.

Answers: **(a)** 60 **(b)(i)** 255 **(ii)** Subtract 180 from their **(b)(i)** **(c)(ii)** 64 **(iii)** 60

Question 5

(a) Most candidates were able to draw pattern 4 correctly on the grid. The most common errors seen included either drawing an extra square in the top row or drawing 2 squares in the bottom row.

(b) (i) Almost all candidates were able to write down the next two terms correctly. The most common errors seen was 17, 23.

(ii) Candidates clearly recognised that the sequence rule was to subtract 6 although candidates did not always express this as required. Whilst minus 6, −6, less 6, were accepted, the *n*th term, 47 − 6*n*, was not accepted or incorrect rules such as *n* − 6 or −6*n* or inaccurate answers such as subtract −6.

(c)(i) Many candidates were able to give the correct expression for the *n*th term. A common acceptable equivalent seen, also awarded full marks, was $11+4(n-1)$. Some candidates were awarded a method mark for $4n+k$. A common incorrect answer was $7n+4$.

(ii) Candidates were able to approach this question in a number of different ways. For full marks candidates needed to give some numerical evidence and support it with some written conclusion. Candidates who solved $4n + 7 = 129$ to reach $n = 30.5$ needed to explain that because 30.5 is not a whole number, 129 cannot be a term in the sequence. Candidates who showed the 30th term is 127 and the 31st term is 131 needed to explain that there is no term between 127 and 131 so 129 cannot be in the sequence. Candidates who equated their equation from part **(c)(i)** to129 were awarded one mark. Some candidates were successful after writing out the first 31 terms in the sequence and giving a supporting explanation, but this was not the recommended approach. Some candidates misread the question and attempted to find the value of the 129th term. Many other approaches were seen and these were awarded marks according to numerical accuracy and the explanation given.

Answers: **(b)(i)** 17, 11 **(ii)** subtract 6 **(c)(i)** 4*n* + 7 **(ii)** no, with correct reason

Question 6

(a) (i) This question was answered accurately by many candidates. Errors included working out 0.66 but then giving a final answer of 0.7 .Candidates who did not score usually did not recognise that they needed to find 3.5×1.24 as a first step.

(ii) Whilst many candidates answered this correctly, there were a significant number of candidates who did not know that 1 kg = 1000 g. Common incorrect answers included 350, 35 and 0.35.

(b) Candidates demonstrated good problem solving skills and careful reading of the question with many correctly giving the maximum number of oranges. Common errors scoring only the method mark included giving an answer of 11.76 or rounding 11.76 to 12. Those candidates giving only an answer of 12 with no supporting working were unable to score as there was no evidence to support where the 12 had come from.

(c) Candidates answered this question well. The most common errors included converting 87% to 0.087 or 8.7 or finding 13% of 700 g.

(d) Candidates answered this question well with an answer correct to at least three significant figures. Some candidates showed no working and an inaccurate answer of 21 with no evidence of where it had come from did not score. Other candidates correctly found 150 but did not go any further. Other common errors included finding the range or median.

(e) A number of candidates were able to write down a pair of correct simultaneous equations and show clear working to solve them, usually by the elimination method. Some candidates were able to write down a pair of equations but could not solve them usually because, after equating coefficients, they added rather than subtracted their equations. Others attempted the elimination method but made errors with signs. A number of candidates were unable to score any marks on this question because they did not know how to form two equations, some using the same letter for both the apples and the plums. One mark was awarded for correct answers with wrong or no working and in addition, method marks were available for those candidates who set up the equations incorrectly but who could demonstrate a correct understanding of the process of solving simultaneous equations.

Answers: **(a)(i)** 0.66 **(ii)** 3500 **(b)** 11 **(c)** 609 **(d)** 21.4 **(e)** 0.3, 0.38

Question 7

(a) Throughout part **(a)**, answers were accepted in fraction, decimal and percentage form. In a question like this it is usually best to give the answer as a fraction and it is not necessary to convert it into another form. Some candidates were unnecessarily giving all 3 forms on their answer lines.

(i) Most candidates answered this part correctly. The most common incorrect answers were $\frac{1}{4}$ or 4.

(ii) Most candidates answered this part correctly. The preferred answer was zero but candidates were rewarded for $\frac{0}{20}$.

(iii) Most candidates answered this part correctly. The most common error was to give the probability of pink, $\frac{3}{20}$.

(b) (i) Candidates were generally able to recognise that this question required Pythagoras' theorem and many were successful in scoring full marks. A number of candidates showed no working and gave the answer of 9.74 which, with no method seen, could not score. The most common incorrect answer was 13.9 from $12^2 + 7^2$.

(ii) Many candidates answered this question correctly and recognised that twice their answer to part **(b)(i)** needed to be added to 14. Follow through marks were awarded a number of times allowing those who did not score in the previous part an opportunity to gain a mark.

(c) (i) This question was not answered well. Candidates need to make sure they read the given information carefully and that they bisect the required angle and the required line. Clear and accurate constructions were required, with all construction arcs evident, for both bisectors.

(ii) For those candidates who had two clear intersecting lines, or the position of the water sprinkler was clearly indicated, a follow through mark was available for 4 times the measured distance on their diagram.

Answers: **(a)(i)** $\frac{4}{20}$ **(ii)** 0 **(iii)** $\frac{17}{20}$ **(iv)** 9.75 **(b)(i)** 33.5 **(c)(ii)** 17.2

Question 8

(a) A large minority of candidates gave the correct mathematical name for line *DE*. The most common incorrect answer was chord. Many candidates did not offer a response.

(b) Many candidates drew a clear radius on the diagram. Some candidates chose to use *OA* or *OB* as their radius and, provided they indicated the end of their radius clearly, they were awarded the mark. Common errors included drawing an ambiguous radius where the intention was not clear or drawing a diameter or chord.

(c) (i) This part was not answered well. Whilst a number of candidates scored a mark for 90°, few candidates explained that the angle between the radius (or diameter) and tangent is 90° or that the radius (or diameter) and tangent are perpendicular.

(ii) Again, whilst a number of candidates knew that angle *ACB* = 90°, few were able to explain clearly that the angle in a semi-circle is 90°. A common incorrect response was to use the word triangle, rather than angle.

(d) (i) This question was answered well with answers frequently given to at least the correct three significant figure accuracy required. Common errors included not giving the correct units or omitting the units or calculating the circumference rather than the area of the circle.

(ii) A number of candidates recognised that trigonometry needed to be used to find the length of *BC*. Some candidates were able to gain the method mark by showing $\sin 35 = \frac{BC}{9}$ even if this was subsequently rearranged incorrectly. Other errors included using $\cos 35 = \frac{BC}{9}$.

Answers: **(a)** tangent **(c)(i)** 90°, radius and tangent **(ii)** 90°, angle in a semi-circle **(d)(i)** 63.6 cm^2 **(ii)** 5.16

Question 9

(a) The majority of candidates completed the table correctly. A common slip was to omit the negative sign from the −1.5 when $x = -4$.

(b) Graphs were generally plotted accurately and smoothly drawn. When points were inaccurately plotted it was usually the (−5, −1.2), (−4, −1.5), (4, 1.5) and (5, 1.2) points because the scale had been consistently misread. Relatively few other errors were seen but included candidates drawing both sides of the graph above or below the *x*-axis, joining the points with a ruler or joining (−1, −6) to (1, 6).

(c) Candidates did well on this question with many correct answers seen including the accurate answer $1\frac{1}{3}$ which had most likely been obtained from solving the equation, even though this was not the intended method. A few candidates were able to obtain the mark from a follow through on their graph. Incorrect answers often came from misreading the scale or from wrongly solving the equation as $6 \times 4.5 = 27$.

(d) (i) Many candidates did not give a response to this question. However, a minority drew a correct ruled line covering the width of the grid. The most common incorrect line seen was $y = -x$.

(ii) A minority of candidates did not give a response to this question. Candidates who had drawn $y = x$ were most successful in this part. Errors came predominantly from misreading the scale or inaccuracies with minus signs.

Answers: **(a)** −1.2, −1.5, 1.5 **(c)** 1.2 to 1.4 **(d)(ii)** (−2.6 to −2.3, −2.6 to −2.3) and (2.3 to 2.6, 2.3 to 2.6)

MATHEMATICS (US)

Paper 0444/43
Paper 43 (Extended)

Key messages

To achieve well in this paper, candidates need to be familiar with all aspects of the extended syllabus.

The recall and application of formulae and mathematical facts in varying situations is required as well as the ability to interpret situations mathematically and problem solve with unstructured questions.

Work should be clearly and concisely expressed with answers written to an appropriate accuracy.

Candidates should show full working with their answers to ensure that method marks are considered.

General comments

The standard of performance was generally good with the vast majority of candidates attempting all questions. Some candidates showed working with stages that could be easily followed. In other cases, candidates omitted some stages or did not show calculations at all.

Some candidates lost marks by approximating values prior to the final answer. This was apparent for example in **Question 6** with angles being rounded to the nearest degree. The requirement for accuracy to the nearest thousand in **Question 2(b)** was often ignored.

The topics that proved to be accessible were reflection, rotation, translation, mid-point of a line, ratio, percentage increase, reverse percentage, exponential increase, curved surface area and volume of a cone, density and mass, plotting points and drawing curves, finding the mean of grouped data, drawing histograms, interpreting a cumulative frequency curve, using the sine rule, probability, simple trigonometry, using functions including composite functions and linear sequences.

More challenging topics included length of a vector, finding annual percentage change for an exponential increase, manipulation of the formulae for volume and surface area of cylinders and spheres, finding the exact answer for the point of intersection of two curves, volumes and surface areas of similar solids and inverse functions.

Candidates appeared to have sufficient time to complete the paper and any omissions were due to lack of familiarity with the topic or difficulty with the question rather than lack of time.

Comments on specific questions

Question 1

(a) Many correct answers were seen. Reflection in either the x-axis or the line $x = -1$ were the two most common incorrect answers. A small number of candidates gave two transformations, reflection in the x-axis followed by a translation of two units down.

(b) Many correct translations were seen with some candidates earning partial credit for a translation with a correct displacement in one direction. Some candidates treated the translation as $\begin{pmatrix} -5 \\ -2 \end{pmatrix}$.

(c) Many correct rotations were seen. Partial credit was earned for a correct 90° rotation either about a wrong centre or in a clockwise direction.

(d) Although a majority of correct enlargements were seen, candidates were generally less successful in this part. Enlargements with scale factor $\frac{1}{2}$ were often seen and, to a lesser extent, enlargements with scale factor $-\frac{1}{2}$ with an incorrect centre. A small number drew enlargements with scale factor 2 and a significant number made no attempt at all.

Answers: **(a)(i)** Reflection $y = -1$

Question 2

(a) (i) Many correct calculations were seen in this part. A common error involved starting with 115 and showing the number of students to be 240. Some candidates did not show both steps in the calculation, usually omitting $240 \div 48$.

(ii) Many correct answers were seen with a significant number earning partial credit for a ratio not in its simplest form, usually 22 : 20. A small number of candidates gave the answer as 10 : 11.

(iii) Almost all candidates gave the correct answer.

(iv) Many candidates understood the method for calculating a reverse percentage. Those that recognised that 240 was 160% of the original number of students almost always went on to obtain the correct answer. A significant number of candidates simply reduced 240 by 60% leading to the common incorrect answer of 96.

(b) A good majority of candidates understood exponential increase and calculated the correct population after 30 years. Some candidates mistakenly calculated the overall increase as 60% and a population of 409600 was a common incorrect answer. Some forgot that the question asked for an answer correct to the nearest thousand, often giving the exact answer instead.

(c) Only a minority of candidates were able to begin correctly with $\left(1+\frac{r}{100}\right)^{32} = 4.09$ or its equivalent. Many of those with a partial understanding did not realise that the population after 32 days was 409% of the original and so 3.09 was usually seen in place of the 4.09. Those that did use 4.09 often went on to find the correct rate of interest. Two errors were very common. Firstly, many divided 309 by 32 and gave an answer of 9.6% and secondly, many calculated $\sqrt[32]{309}$ which led to an answer of 19.6%.

Answers: **(a)(ii)** 11 : 10 **(iii)** 276 **(iv)** 150 **(b)** 464000 **(c)** 4.50

Question 3

(a) (i) Most candidates calculated the correct curved surface area. Common errors usually involved using the perpendicular height or calculation of the total surface area.

(ii) Again, most candidates were able to calculate the correct volume of the cone. Common errors usually involved the incorrect use of Pythagoras' theorem, often $17^2 + 8^2$, and the occasional use of 17 for the height of the cone.

(iii) Almost all candidates demonstrated a good understanding and calculated a correct mass for their volume in the previous part. The most common error was dividing the volume by 0.8.

(iv) The vast majority of candidates calculated the correct mass of the box. Adding the mass of the cone to 1.2 kg and confusion over the number of grams in a kilogram were the most common errors.

(b) (i) Although many candidates were able to make a start, only a minority were able to obtain the correct fraction. The manipulation and simplification of the algebraic formulae proved to be challenging. For the cylinder, squaring $3r$ incorrectly to give $3r^2$ was a very common error. Cancelling the fraction but still having π or r in the fraction or inverting the fraction were also common errors. The absence of numerical values to substitute into the two formulae caused problems. Some overcame this by allocating a value to r. Rounding intermediate values during the process meant that candidates rarely reached the correct final fraction.

(ii) A majority of candidates understood the need to find the radius from the given information. Equating $4\pi r^2$ and 81 was a common error at this stage. With $r = 4.5$ many were able to find the correct curved surface area. Other common errors included the use of r as the radius instead of $3r$ and squaring $3r$ incorrectly. Some forgot that the question asked for the answer to be given in terms of π and answers in the range 3052 to 3055 were seen. Some attempted to convert their numerical answer to a multiple of π but this rarely gave the exact multiple.

Answers: **(a)(i)** 427 **(ii)** 1010 **(iii)** 808 **(iv)** 392 **(b)(i)** $\frac{1}{54}$ **(ii)** 972π

Question 4

(a) Almost all candidates completed the table correctly.

(b) Many candidates plotted the points correctly and drew an acceptable curve. Plotting (3, 0.9) at (3, –0.9) was a common error. Although many good curves were seen, candidates need to understand that a curve should be drawn with a sharp pencil. Some used a pen which led to problems when corrections had to be made and others used blunt pencils that produced curves that were far too thick. However, very few candidates joined up the points with ruled lines.

(c) A majority of candidates were able to draw an accurate tangent and calculate its gradient correctly. Some candidates calculated change in x divided by change in y. Others misread the scales when finding the changes in x and y and some made numerical slips, largely because of the negative values used. Having done all the correct work some ignored the negative and gave the absolute value of the gradient. Some less able candidates made no attempt at this part.

(d) Almost all candidates completed the table correctly.

(e) Many candidates plotted the points correctly and drew acceptable curves with very few instances where candidates had joined the two separate sections. As in part **(b)** some used a pen and some drew curves that were far too thick. Again, very few joined up the points with ruled lines.

(f) (i) A majority of candidates gave a correct value for x at the point where the curves intersected.

(ii) This proved challenging for many candidates and only a minority were able to use their previous answer to give a correct inequality. Some did use their answer but then used less than instead of greater than. Many incorrect responses did not use their previous answer.

(g) Again this proved challenging. Only a minority were able to set up a correct equation, rearrange it and find the correct value of k. Most did not use algebra, preferring to cube the value from part **(f)(i)**.

Answers: **(a)** –1, 3 **(c)** –2 to –1.5 **(d)** –3, 3 **(f)(i)** 3.6 to 3.85 **(ii)** $x >$ *their* **f(i)** **(g)** 52

Question 5

(a) i) Calculating the mean of grouped data was well answered with most candidates showing clear and accurate working leading to the correct answer. Only a small minority of incorrect answers resulted from the use of the interval widths.

(ii) Many accurately drawn histograms were seen, the majority of which were neatly ruled. Calculations for the frequency densities were rarely seen and if blocks were incorrect it was often impossible to spot where the candidates had gone wrong. Drawing the first block to cover the width from 0 to 200 tonnes was a common error.

(b) (i) Many correct answers were seen. Answers of 8 and 10 were two common errors but there was no pattern to many of the other incorrect answers.

(ii) The median was usually stated correctly.

(iii) The value of the upper quartile was usually correct.

(iv) Although many candidates obtained the correct value for the interquartile range, fewer correct answers were seen than in the previous parts. Some gave incorrect answers such as 38 – 62, others gave the value of the lower quartile.

(v) Many candidates were able to estimate the number of days when the mass was greater than 20 tonnes. Occasionally candidates stated the number of days when the mass was less than 20 tonnes. Some candidates misread the scale and incorrect answers of 89 and 11 were seen.

Answers: **(a)(i)** 265 **(b)(i)** 100 **(ii)** 56 **(iii)** 62 **(iv)** 24 **(v)** 88

Question 6

(a) A majority of candidates showed their working clearly and obtained the correct value for the angle. Some started with $11^2 = 13^2 + 4^2 - 2 \times 13 \times 4 \times \cos C$ and then rearranged it incorrectly trying to obtain an expression for cos*C*. Others had a correct statement of the cosine rule for an angle other than *C* and some stated the cosine rule incorrectly. Many of the less able candidates found this question challenging and did not recognise that the cosine rule was needed and the use of the sine rule and simple trigonometry were seen.

(b) Most candidates calculated angle *DAC* correctly. Some rounded their answer to 37 which led to their answer for angle *ACD* being out of range. A significant number of candidates recognised that the sine rule could be used directly for angle *ACD* if the length of *AD* could be found. Some candidates set up the cosine rule for angle *D* correctly which led to a quadratic equation in *AD* which was sometimes solved correctly. Many of the other attempts to find *AD* were incorrect, such as Pythagoras' theorem and simple trigonometry.

(c) Full correct answers were in the minority although many candidates earned partial credit for a correct method for the area using their incorrect angles from the previous two parts. Several candidates successfully calculated a correct perpendicular height for each triangle and used them correctly. Some used the wrong angle, not realising that the angle in the area formula needed to be the included angle. Some treated one or both triangles as right-angled triangles for a simpler calculation.

Answers: **(a)** 52.0 **(b)** 62.7 **(c)** 66.7

Question 7

(a) (i) Many correct responses were seen. Having found the correct probabilities, a minority of candidates made no comparison which required using an inequality sign, finding equivalent fractions with the same denominator or converting to a decimal or percentage.

(ii) A majority of candidates had a good understanding of probability and were able to use a correct method for calculating the probability. Several attempted a tree diagram with some success but not all trees showed all of the correct outcomes A common error occurred in the fraction work with a significant number of candidates thinking that $\frac{2}{5} \times \frac{1}{4} = \frac{3}{20}$. Some applied the wrong operations and $\left(\frac{3}{5} + \frac{3}{4}\right) \times \left(\frac{2}{5} + \frac{1}{4}\right)$ was often seen. Others attempted to calculate the probability of two different colours from each bag.

(b) (i) Many correct answers were seen in this part. Some calculated the probability with replacement and some appeared to refer back to the previous part and calculated the probability that both balls were black.

(ii) Although many correct answers were seen, candidates were slightly less successful in this part. Some thought that the probability could be found by using 1 – probability of all black. Others gave a probability with replacement and some ignored the fact that there were not three white balls in the bag and gave the answer as $\frac{2}{5} \times \frac{1}{4}$.

(c) Although this proved to be a challenging question, a small majority of candidates were able to calculate the correct probability. The fact that the ball chosen from bag *A* was placed in bag *B* did not fully register with all candidates as many still gave their probabilities out of 4. An error similar to the one in part **(a)(ii)**, $\frac{2}{5} \times \frac{1}{5} = \frac{3}{25}$, was often seen. A small number calculated the probability of the two balls having the same colour but not all subtracted their answer from 1.

Answers: **(a)(ii)** $\frac{11}{20}$ **(b)(i)** $\frac{6}{60}$ **(ii)** 0 **(c)** $\frac{11}{25}$

Question 8

(a) (i) Many correct answers were seen in this part. Some could not identify the corresponding sides correctly and 3 was the most common error.

(ii) Most candidates had a good understanding of trigonometry and had no difficulty in finding angle *XAB*. Some calculated angle *XBA* as their first step and some used the cosine rule. Some candidates lost accuracy by giving their answer as 37 without first showing a more accurate value.

(b) This part proved to be quite demanding for many of the candidates. With the complexity of the diagram, candidates needed to look carefully in order to identify the circle theorems that could be applied to the work. Identifying the value of *v* was the key part of the question and most of the more able candidates found *v* correctly and followed this with the remaining three angles. Many of the rest did not identify *v* as the angle at the centre and the 75 as the matching angle at the circumference. More candidates recognised that triangle *OPT* was isosceles and so were able to find *w* correctly from their *v*. Not all candidates recognised *w* and *x* as a pair of angles at the circumference. Many earned partial credit for *y* by identifying the third angle in the triangle with *y* and *v* (or its opposite) as 20°.

(c) To achieve success in this question, candidates needed a good understanding of scale factors for length, area and volume and how to use them. Only the more able candidates achieved much success and fully correct responses were in the minority. Some recognised that $\frac{94}{226}$ was an area scale factor and were able to square root it to find the linear scale factor. Many others simply used the area factor as the volume factor and obtained an answer of 282, a very common incorrect answer.

Answers: **(a)(i)** 4 **(ii)** 36.9 **(b)** 150 15 15 10 **(c)** 182

Question 9

(a) Many correct column vectors were seen with only a few examples of the components separated by a fraction line. The most common incorrect answers included two by two matrices that used the co-ordinates as their elements.

(b) The process of finding the magnitude of a vector was not understood by many of the candidates and only a minority gave a correct value. There was no pattern to the wide variety of incorrect answers. Some of the more common ones included giving the equation of the line *AB*, adding or multiplying the components of the vector and some just repeated the vector. A significant number of blank responses were seen.

(c) Many correct answers were seen. Common errors included doubling the co-ordinates of *B* and finding the mid-point of *AB*.

(d) A majority of candidates clearly understood the method required, showed the various stages of their working and obtained the correct equation. Having found the correct gradient, a significant number went on to find the gradient of the normal and attempt to find its equation. Many of the incorrect equations resulted from slips when calculating the gradient. With or without the correct gradient, not all candidates appreciated that the co-ordinates of a point on the line could be substituted to find the constant term. Only a small number of candidates used *A* with either *B* or the mid-point to count back to find the intercept.

(e) A majority of candidates earned credit for the correct answer or for an answer that followed through from their incorrect equation. Not all candidates with the correct equation gave the correct co-ordinates of *D* as several gave the answer as (–4, 0). Others gave the co-ordinates of the point where the line crossed the *x*-axis.

Answers: **(a)** $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ **(b)** 4.47 **(c)** (7, 10) **(d)** $y = 2x - 4$ **(e)** (0, –4)

Question 10

(a) The vast majority of candidates calculated the value of the function correctly.

(b) Many correct responses were seen in this part. Most candidates evaluated h(–1) as their first step and then substituted its value into the function f(x). Some opted to find the composite function fh(x) as their first step but this usually produced more errors, such as $3(3^x) + 4 = 9^x + 12$. Some found the product of the two functions.

(c) Finding the inverse of a linear function was well answered. Sign errors were common when rearranging the terms, particularly from less able candidates. A number of candidates did not earn full credit as they left an otherwise correct answer in terms of *y*.

(d) The composition of two functions, ff(x), was more challenging but many candidates answered this part well. Many of the less able candidates took the composition to be a product of the two functions, resulting in a quadratic expression that replicated the next part if carried out correctly. Errors with the expansion of brackets and the resulting simplification were seen quite often

(e) Squaring the function f(x) was completed correctly by many candidates. Errors with the expansion of brackets and the resulting simplification were quite common. Less able candidates often squared the individual terms and the answer $9x^2 + 16$ was quite common.

(f) Success in this question was largely dependent on the algebraic step of moving from $h^{-1}(x) = g(2)$ to the composite function $x = hg(2)$. This was rarely seen and only a small minority obtained the correct answer. The log function was used in a very small number of cases with limited success. The common error was to equate 3^x to g(2). Several candidates treated the inverse function as the reciprocal function.

Answers: **(a)** 0 **(b)** 5 **(c)** $\frac{x+1}{2}$ **(d)** $9x + 16$ **(e)** $9x^2 + 24x + 16$ **(f)** 27

Question 11

(a) Most candidates obtained the next term in the sequence. Those that treated the numerators and denominators as separate sequences usually had no difficulty in finding an expression for the *n*th term. When errors occurred, they were more likely to be seen in the denominator. Those that worked with the fractions as a whole, made no progress as they could find no common difference. A significant number of candidates made no attempt at the *n*th term.

(b) (i) Many correct responses were seen with most in their simplest form. Common incorrect answers included $n-2$, the next term and the common difference.

(ii) Many candidates produced a difference table showing the third difference to be 6. Some of these led to a cubic expression but some led to linear expressions involving $6n$ or quadratic expressions involving $6n^2$ or $3n^2$. Some candidates with the correct expression did so without showing working. There was a significant number of blank responses.

Answers: **(a)** $\frac{8}{15}$, $\frac{n+2}{2n+3}$ **(b)(i)** $1-2n$ **(ii)** n^3+1